



Conceptions for heat transfer correlation of nanofluids

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Abstract

The nanofluid is a solid–liquid mixture in which metallic or nonmetallic nanoparticles are suspended. The suspended ultrafine particles change transport properties and heat transfer performance of the nanofluid, which exhibits a great potential in enhancing heat transfer. The mechanism of heat transfer enhancement of the nanofluid is investigated. Based on the assumption that the nanofluid behaves more like a fluid rather than a conventional solid–fluid mixture, this article proposes two different approaches for deriving heat transfer correlation of the nanofluid. The effects of transport properties of the nanofluid and thermal dispersion are included. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The term “nanofluid” is envisioned to describe a solid–liquid mixture which consists of nanoparticles and a base liquid, and which is one of the new challenges for thermo-science provided by the nano-technology. There are several approaches to prepare nanofluids. Some auxiliary activators or dispersants are necessary to obtain the even distributed and stabilized suspensions. The ultrafine particles may be either metallic or nonmetallic. In general, the nanofluids used for the purpose of enhanced heat transfer are dilute multicomponent fluids and the volume fractions of nanoparticles are below 5–10%. The nanoparticles suspended in the nanofluids can be the nanostructured

materials below 100 nm in diameter. Heat transfer performance of the nanofluid is superior to that of the original pure fluid because the suspended ultrafine particles remarkably increase the thermal conductivity of the mixture and improve its capability of energy exchange. Several literature [1–3] reveal that with low nanoparticles concentrations (1–5 vol %), the thermal conductivity of the suspensions can increase by more than 20%. Such enhancement mainly depends upon factors such as the shape of particles, the dimensions of particles, the volume fractions of particles in the suspensions, and the thermal properties of particle materials.

The nanofluids have a unique feature which is quite different from those of the conventional solid–liquid mixtures in which millimeter and/or micrometer-sized particles are added. As we know, the particles of millimeter or micrometer magnitudes settle rapidly, clog flow channels, erode pipelines, and cause severe pressure drops, etc. All these shortages limits the application of the conventional

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Nomenclature

c_p specific heat capacity
 d_p diameter of particles
 h heat transfer coefficient
 k thermal conductivity
 Nu Nusselt number
 Pr Prandtl number
 R radius of a tube
 Re Reynolds number
 q heat flux
 T temperature
 u velocity
 V volume
 r radial variable
 x axial variable

α thermal diffusivity
 ϕ volume fraction of the nanoparticles
 μ viscosity
 ρ density

Subscripts

d dispersion effect
 f fluid
 m mean value
 nf nanofluid
 r radial direction
 s solid
 x axial direction

solid–liquid mixture. The concept of the nanofluid is an innovative idea [4], although the idea about solid–liquid suspensions appeared a long time ago. Modern technology makes it possible to produce a variety of stabilized nanofluids. Because of their excellent characteristics, the nanofluids find wide applications in enhancing heat transfer, even for microscale heat transfer [5]. It is expected that the nanofluid will become a new type of heat transfer fluid for thermal engineering.

Compared with the existing techniques for enhancing heat transfer, the nanofluids show a great potential in increasing heat transfer rates in a variety of application cases, with incurring either little or no penalty in pressure drop. Although the nanofluids have great potential for enhancing heat transfer, research work on the concept, enhancement mechanism, and application of the nanofluids is still in the primary stage. Several existing published articles are mainly focused on prediction and measurement techniques of thermal conductivity of the nanofluids. To our knowledge, there are only few references involved in describing heat transfer performance of the nanofluids. A complete understanding about the heat transfer performance of the nanofluids is necessary for their practical application to heat transfer enhancement. In nature, the nanofluids are multicomponent fluids, even the dispersed particles are ultrafine. Three fundamental motions of multicomponent materials, sedimentation, shearing flow, and wave motion, may coexist in flow of the nanofluids. It is difficult to establish any formulated theory that could predict the flow of a dispersed multicomponent material [6]. Such an approach may

also be impracticable for practical application of the nanofluids to heat transfer enhancement.

It is expected that heat transfer coefficient (Nusselt number) of the nanofluid depends upon a number of factors such as thermal conductivity and heat capacity of both the base fluid and the ultrafine particles, the flow pattern, the viscosity of the nanofluid, the volume fraction of the suspended particles, the dimensions and the shape of these particles as well as on the flow structure. Therefore, the general form of the Nusselt number yields

$$Nu_{nf} = f \left(Re, Pr, \frac{k_s}{k_f}, \frac{(\rho c_p)_s}{(\rho c_p)_f}, \phi, \right. \\ \left. \text{dimensions and shape of particles, flow structure} \right) \quad (1)$$

So far we have found no published literature on deriving the convective heat transfer correlation of the nanofluid from either theoretical or experimental approach.

The purpose of this article is to analyze heat transfer performance of the nanofluids and to derive some fundamental correlations for predicting convective heat transfer of the nanofluids with two different approaches. One is the conventional way to treat the nanofluids as the single-phase fluids and another is to account for the multiphase feature of the nanofluid and the dispersed nanoparticles.

2. The conventional approach

From various sources, one may find a variety of convective heat transfer correlations of single-phase

flow for different application cases. Whether these relations are applicable for predicting heat transfer performance of the nanofluids invokes a researchers' interest. If the answer is positive, it will be convenient to extend the available correlations of the conventional single-phase fluids to the corresponding applications of the nanofluids since there are no published correlations for the nanofluid application. On certain assumptions, such extension may be feasible. The nanofluids used for the purpose of heat transfer enhancement are usually dilute solid–liquid mixtures. Since the solid particles are ultrafine (< 100 nm) and they are easily fluidized, these particles can be approximately considered to behave like a fluid. Under the assumptions that there exist no motion slip between the discontinuous phase of the dispersed ultrafine particles and the continuous liquid and the local thermal equilibrium between the nanoparticles and the fluid, the nanofluid can be treated as the common pure fluid. All the equations of continuity, motion, and energy for the pure fluid are directly extended to the nanofluid. Under the assumption of constant thermal properties, for example, the energy equation for the incompressible flow of a pure fluid without viscous dissipation

$$\frac{\partial T}{\partial t} + \nabla \cdot \mathbf{u}T = \nabla \cdot (\alpha_f \nabla T) \quad (2)$$

is also suitable to describe the heat transfer process of the nanofluid. It means that the solutions for the single-phase fluid is also valid for the nanofluid in the identical application cases. However, it must be emphasized that the thermal properties appearing in Eq. (2) refers to those of the nanofluid.

Thus, the dimensionless correlations of heat transfer of the pure fluid are applicable for the nanofluid. For example, the following relations

$$Nu = 3.66 \quad (3a)$$

(fully developed laminar flow, tube, $T_w = \text{const}$)

$$Nu = 0.023 Re^{0.8} Pr^{1/3} \quad (\text{turbulent flow, tube}) \quad (3b)$$

can be used to calculate heat transfer rates of the nanofluid flowing in a tube in the laminar region and turbulent region, respectively. A similar approach was introduced by Nield and Bejan [7] for comparing the dimensionless expressions of fully developed heat transfer in a channel without and with a porous matrix. However, one must pay attention in selecting the suitable thermal properties and transport properties. While applying the existing dimensionless relations for pure fluids to the nanofluid, one needs to use the properties corresponding to the nanofluid. Three main parameters involved in calculating heat transfer rate of

the nanofluid are heat capacity, viscosity, and thermal conductivity, which may be quite different from those of the original pure fluid. For the synthesized nanoparticle–liquid suspension, the parameter $(\rho c_p)_{\text{nf}}$ of the nanofluid is expressed as

$$(\rho c_p)_{\text{nf}} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s \quad (4)$$

The viscosity of the nanofluid can be estimated with the existing relations for the two-phase mixture. Drew and Passman [6] introduced the well-known Einstein's formula for evaluating the effective viscosity μ_{eff} of a linearly viscous fluid of viscosity μ_f containing a dilute suspension of small rigid spherical particles. The formula yields:

$$\mu_{\text{eff}} = \mu_f(1 + 2.5\phi) \quad (5)$$

This relation is restricted for low volume concentration ($\phi < 0.05$). Einstein's equation was extended by Brinkman [8] as

$$\mu_{\text{eff}} = \mu_f \frac{1}{(1 - \phi)^{2.5}} \quad (6)$$

One may find other relations of the effective viscosity of the two-phase mixture in the literature [6,9]. Each relation has its own application limitation. The direct and reliable access to obtaining the apparent viscosity of the nanofluid is by experiment. Xuan and Li [10] have experimentally measured the apparent viscosity of the transform oil–water nanofluid and of the water–copper nanofluid in the temperature range of 20–50°C. The experimental results reveal relatively good coincidence with Brinkman's theory.

The apparent thermal conductivity is the most important parameter to indicate the enhancement potential of the nanoparticle–liquid suspension. Research has shown that the thermal conductivity of the nanofluid is a function of thermal conductivity of both the base fluid and the nanoparticle material, the volume fraction, the surface area, and the shape of the nanoparticles suspended in the liquid, and distribution of the dispersed particles. There are no theoretical formulas available yet for predicting the thermal conductivity of nanofluids. Some existing theoretical models for thermal conductivity were proposed for the solid–liquid mixtures with relatively large particles of the order of micrometers or millimeters, in which the apparent thermal conductivity of the suspensions depends only upon the volume fraction and shape of the suspended particles, not upon the size and distribution of the particles. Application of these models to the nanofluids is limited. One should experimentally determine the thermal conductivity of the nanofluids. The transient hot-wire method can be adopted for this purpose [1–3].

In the absence of experimental data and suitable theory for the thermal conductivity of the nanofluid, some existing formulas for predicting the thermal conductivity of solid–liquid suspensions with relatively larger particles may be extended approximately to estimate that of the nanofluid. For solid–liquid mixtures in which the ratio of conductivity of two phases is larger than 100, Hamilton and Crosser [11] developed the following model:

$$\frac{k_{\text{eff}}}{k_f} = \frac{k_p + (n-1)k_f - (n-1)\phi(k_f - k_p)}{k_p + (n-1)k_f + \phi(k_f - k_p)} \quad (7)$$

where the empirical shape factor given by

$$n = \frac{3}{\psi} \quad (8)$$

where ψ is the sphericity, defined as the ratio of the surface area of a sphere with a volume equal to that of the particle to the surface area of the particle. Their experimental results showed satisfactory coincidence between the theoretical predictions and the experimental data for spherical particles in the range of volume fraction up to 30%. For particles of other shapes, the shape factor n can be allowed to vary from 0.5 to 6.0.

An alternative expression for calculating the effective thermal conductivity of solid–liquid mixtures was introduced by Wasp [12]:

$$\frac{k_{\text{eff}}}{k_f} = \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)} \quad (9)$$

Comparison between these two expressions reveals that the latter is a special case with the sphericity 1.0 of the former.

The Hamilton–Crosser model was used by Xuan and Li [3] to obtain a rough estimation of the thermal conductivity of the nanofluids with different values of ψ from 0.5 to 1.0 and the results from the model corresponding to $\psi = 0.7$ are close to their experimental data. Lee et al. [1] pointed out that the predicted thermal conductivity ratios for spherical particles ($\psi = 1$) from this model are in good agreement with their experimental results of Al_2O_3 nanofluids. Thus, this model may be used for approximation. However, it must be emphasized that such a primitive estimation gives no warranty and the accurate and reliable formulas as well as experimental data are needed for determining the effective thermal conductivity of the nanofluid.

According to the above description, one learns that the conventional approach of finding heat transfer coefficient of the nanofluids is through a method in which the existing heat transfer coefficient correlations for the pure fluid are directly extended to the nano-

fluids, with substituting the thermal properties and transport parameters of the nanofluids for those of the pure fluids. Here a superficial conclusion could be that heat transfer enhancement of nanofluids is realized by increasing the effective thermal conductivity of nanofluids. The augmentation effect of heat transfer can approximately be indicated by the ratio

$$\frac{h_{\text{nf}}(\text{nanofluid})}{h_f(\text{base fluid})} \sim \left(\frac{k_{\text{nf}}}{k_f}\right)^c \quad (10)$$

where the exponent c is a constant which depends on the flow pattern. For example, $c = 2/3$ for the turbulent flow. By means of this formula, one can approximately estimate heat transfer enhancement of the nanofluid by enlargement of its thermal conductivity.

3. The modified conventional approach

Although the nanofluid behaves more like a fluid than the conventional solid–fluid mixtures in which relatively larger particles with micrometer or millimeter orders are suspended, it is a two-phase fluid in nature and has some common features of the solid–fluid mixtures. In view of either microscale or macroscale, however, it may be questionable whether the theory of the conventional two-phase flow can be applied in describing the flow characteristics of the nanofluid. Because of the effects of several factors such as gravity, Brownian force, and friction force between the fluid and ultrafine solid particles, the phenomena of Brownian diffusion, sedimentation, dispersion may coexist in the main flow of a nanofluid. This means that the slip velocity between the fluid and the particles may not be zero, although the particles are ultrafine. Irregular and random movement of the particles increases the energy exchange rates in the fluid, i.e., thermal dispersion takes place in the flow of the nanofluid. The thermal dispersion will flatten the temperature distribution and make the temperature gradient between the fluid and wall steeper, which augments heat transfer rate between the fluid and the wall. Therefore, the enhancement mechanism of heat transfer by the nanofluid can be explained from the following two aspects: one is that the suspended particles increase the thermal conductivity of the two-phase mixture and another is that the chaotic movement of the ultrafine particles, the thermal dispersion, accelerates the energy exchange process in the fluid. There is no question that the thermal dispersion plays an important role in heat transfer enhancement. In their work on the dispersed solid–liquid two-phase flow, Sohn and Chen [13] treated the solid–fluid mixture as a fluid to investigate microconvective thermal conductivity in two-phase mixtures and proposed a formula in which the total thermal conduc-

tivity was treated as a function of the Peclet number. The particles of 0.3 mm diameter were used in their experiment.

The aforementioned approach takes the first factor into account, but neglects the second. To account for both the factors, the dispersion model can be adopted. Assume that irregular movement of the ultrafine particles induce small perturbations of both the temperature and velocity of the nanofluid, i.e., T' and \mathbf{u}' , respectively. Thus, the intrinsic phase averages are given as

$$T = \langle T \rangle^f + T' \tag{11a}$$

$$\mathbf{u} = \langle \mathbf{u} \rangle^f + \mathbf{u}' \tag{11b}$$

where one has $\langle T \rangle^f = \frac{1}{V_f} \int_{V_f} T dV$, $\langle \mathbf{u} \rangle^f = \frac{1}{V_f} \int_{V_f} \mathbf{u} dV$, $\frac{1}{V_f} \int_{V_f} T' dV = 0$. In the light of the procedure described by Kaviany [14] and by assuming that the boundary surface between the fluid and the particles is so small that can be neglected, one obtains the following expression from Eq. (2)

$$(\rho c_p)_{nf} \left[\frac{\partial \langle T \rangle^f}{\partial t} + \langle \mathbf{u} \rangle^f \cdot \nabla \langle T \rangle^f \right] = \nabla \cdot (k_{nf} \nabla \langle T \rangle^f) - (\rho c_p)_{nf} \nabla \langle \mathbf{u}' T' \rangle^f \tag{12}$$

The second term on the right-hand side of Eq. (12) indicates the effect of the thermal dispersion resulting from chaotic movement of the nanoparticles in the flow. By means of an analogy with the treatment of turbulence, the heat flux induced by the thermal dispersion in nanofluid flow can be expressed as

$$q_d = (\rho c_p)_{nf} \langle \mathbf{u}' T' \rangle^f = -\mathbf{k}_d \cdot \nabla \langle T \rangle^f \tag{13}$$

where \mathbf{k}_d is the tensor of the dispersed thermal conductivity. Eq. (12) can be rewritten as

$$\frac{\partial \langle T \rangle^f}{\partial t} + \langle \mathbf{u} \rangle^f \cdot \nabla \langle T \rangle^f = \nabla \cdot \left[\left(\alpha_{nf} \mathbf{I} + \frac{\mathbf{k}_d}{(\rho c_p)_{nf}} \right) \cdot \nabla \langle T \rangle^f \right] \tag{14}$$

Then, the energy balance equation for the nanofluid can be given as

$$(\rho c_p)_{nf} \frac{\partial \langle T \rangle}{\partial t} + (\rho c_p)_{nf} \langle \mathbf{u} \rangle \cdot \nabla \langle T \rangle = (\rho c_p)_{nf} \nabla \cdot (\mathbf{D} \cdot \nabla \langle T \rangle) \tag{15}$$

where \mathbf{D} may be called as the total effective thermal diffusivity tensor which includes both the molecular effect and the effect of the thermal dispersion. Here

$$\mathbf{D} = \frac{k_{nf} \mathbf{I}}{(\rho c_p)_{nf}} + (1 - \phi) \mathbf{D}_d \tag{16}$$

It is expected that the thermal dispersion tensor is a function of the flow pattern, properties of both the base fluid and the nanoparticles, the dimensions and shape of the nanoparticles, and the volume fraction of the nanoparticles suspended in the mixture.

For the flow of the nanofluid inside a tube, the energy equation may be simplified as

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[\left(\alpha_{nf} + \frac{k_{d,r}}{(\rho c_p)_{nf}} \right) r \frac{\partial T}{\partial r} \right] \tag{17}$$

with accounting for the thermal dispersion in the radial direction or

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[\left(\alpha_{nf} + \frac{k_{d,r}}{(\rho c_p)_{nf}} \right) r \frac{\partial T}{\partial r} \right] + \frac{\partial}{\partial x} \left[\left(\alpha_{nf} + \frac{k_{d,x}}{(\rho c_p)_{nf}} \right) \frac{\partial T}{\partial x} \right] \tag{18}$$

with the thermal dispersion in both the radial and axial directions. Here the radial and axial dispersed thermal conductivity may be different from each other. With respect to the flow of salt-solution in a tube, Aris [15] and Taylor [16] approximately derived the thermal diffusivity coefficients for the laminar and turbulent flow, respectively,

$$\frac{D_d}{\alpha_f} = \frac{1}{48} Pe^2 \quad \text{for the laminar flow} \tag{19a}$$

$$\frac{D_d}{\alpha_f} = 7.14 Pe \gamma^{1/2} \quad \text{for the turbulent flow} \tag{19b}$$

where the Peclet number Pe is defined as $Pe = Ru_m/\alpha_f$ and $\gamma(Re)$ is the resistance coefficient. The Peclet number Pe comprehends the effects of the macroscale convective and microscale molecular diffusion.

So far there is neither theoretical nor experimental work published on the thermal diffusivity coefficients of the nanofluid. By reference to some publications [17,18] about the thermal dispersion in porous media, it is assumed that the dispersed thermal conductivity of the nanofluid may take the following form:

$$k_d = C(\rho c_p)_{nf} u d_p R \phi \quad \text{or} \quad k_d = C^*(\rho c_p)_{nf} u R \tag{20}$$

where C or C^* is an unknown constant which should be determined by matching experimental data. Other forms of the dispersed thermal conductivity are also possible, which may vary from case to case. To take the effect of thermal dispersion into account, the surface heat flux can be expressed as

$$q = -\left(k_{\text{nf}} \frac{\partial T}{\partial r} + k_{\text{d},r} \frac{\partial T}{\partial r}\right)_{r=R} \quad (21)$$

This expression is for the flow in a tube. Similar to the procedure introduced by Nield and Bejan [7] in their book, the dimensionless analysis of Eq. (21) formally results in the heat transfer correlation of the nanofluid as

$$Nu_x = [1 + C^* Pe^n f'(0)] \theta'(0) Re^m \quad (22a)$$

and with regard to the flow pattern, it may be modified as

$$Nu_x = [1 + C^* Pe^n f'(0)] \theta'(0) Re^m \quad (22b)$$

where $Pe = Ru_m/\alpha_{\text{nf}}$, the exponent m and n depend upon the flow pattern, and the dimensionless velocity f and dimensionless temperature θ may be defined according to the case of application. f' and θ' are the derivatives of the dimensionless velocity and the dimensionless temperature, respectively. Thus, the thermal dispersion increases heat transfer. What needs to be emphasized is that experiment is necessary to determine the unknown coefficient C^* . The case $C^* = 0$ corresponds to zero thermal dispersion. This expression clearly indicates that heat transfer enhancement of the nanofluid increases with the Peclet number. For the nanofluid flow over a flat plate or other forms, the Nusselt correlations similar to Eq. (22) can be derived.

4. Discussions

The nanoparticles enhance heat transfer rate by increasing the thermal conductivity of the nanofluid and incurring thermal dispersion in the flow, which is an innovative way of augmenting heat transfer process. Although there are some sophisticated theories as well as correlations for the conventional solid–fluid flows [6,19–21], it is questionable and doubtful that these theories and correlations are applicable to the nanofluid. As a new type of heat transfer medium, the nanofluid behaves more like a single-phase fluid because the discontinuous phase consists of ultrafine particles, so that the heat transfer correlation for the single-phase fluid, rather than those for the conventional solid–liquid two-phase flow, has been taken as the start point. By starting from the existing theory and correlations for the heat transfer process of pure fluids, this paper has proposed two approaches of deriving heat transfer correlation for the nanofluid. To handle the thermal dispersion resulting from irregular movement of the nanoparticles, the dispersion model has been used.

The nanofluid appears with development of

nanoscience and nanotechnology. The great potential of nanofluids in enhancing heat transfer means chances and challenges in thermal science and engineering. Since the concept of the nanofluid is newly proposed, there are number of questions which remain unclear and need to be solved. Besides the above-mentioned movement modes of the ultrafine particles, for example, aggregation may take place and clusters may be formed in the flow of the nanofluid. All these chaotic movement modes of the nanoparticles will affect the distribution of the particles, transport properties and heat transfer performance of the nanofluid. Research on microscale is necessary to learn the microstructure of the nanofluid, which would help in understanding the flow and heat transfer process of the nanofluid. Theoretical and experimental research is needed in order to apply the nanofluid for the enhancement of heat transfer and to assess the effects by its use.

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References

- [1] S. Lee, S.U.-S. Choi, S. Li, J.A. Eastman, Measuring thermal conductivity of fluids containing oxide nanoparticles, *J. of Heat Transfer* 121 (1999) 280–289.
- [2] H. Masuda, A. Ebata, K. Teramae, N. Hishinuma, Alternation of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles (dispersion of γ - Al_2O_3 , SiO_2 and TiO_2 ultra-fine particles), *Netsu Bussei (Japan)* 4 (1993) 227–233.
- [3] Y. Xuan, Q. Li, Heat transfer enhancement of nanofluids, *Int. J. Heat Fluid Flow* 21 (2000), 158–64.
- [4] S.U.-S. Choi, Enhancing thermal conductivity of fluids with nanoparticles, *Developments and applications of non-Newtonian flows*, ASME FED 231/MD 66 (1995) 99–103.
- [5] S. Lee, S.U.-S. Choi, Application of metallic nanoparticle suspensions in advanced cooling systems, in: *Recent Advances in Solids/Structures and Application of Metallic Materials*, ASME PVP 342/MD 72 (1996) 227–234.
- [6] D.A. Drew, S.L. Passman, *Theory of Multicomponent Fluids*, Springer, Berlin, 1999.
- [7] D.A. Nield, A. Bejan, *Convection in Porous Media*, 2nd ed., Springer, Berlin, 1992.
- [8] H.C. Brinkman, The Viscosity of concentrated suspensions and solutions, *J. Chemistry Physics* 20 (1952) 571–581.
- [9] N. Zuber, On the dispersed two-phase flow in the lami-

- nar flow regime, *Chemical Engineering Science* 19 (1964) 897–917.
- [10] Y. Xuan, Q. Li, Experimental research on the viscosity of nanofluids, Report of Nanjing University of Science and Technology (in Chinese), 1999.
- [11] R.L. Hamilton, O.K. Crosser, Thermal conductivity of heterogeneous two-component systems, *I&EC Fundamentals* 1 (1962) 182–191.
- [12] F.J. Wasp, *Solid–Liquid Slurry Pipeline Transportation*, Trans. Tech, Berlin, 1977.
- [13] C.W. Sohn, M.M. Chen, Microconvective thermal conductivity in disperse two-phase mixture as observed in a low velocity couette flow experiment, *J. of Heat Transfer* 103 (1981) 47–51.
- [14] M. Kaviany, *Principles of Heat Transfer in Porous Media*, Springer, Berlin, 1995.
- [15] R. Aris, On the dispersion of a solute in a fluid flowing through a tube, *Proc. Roy. Soc. (London)* A235 (1956) 67–77.
- [16] G.I. Taylor, The dispersion of matter in turbulent flow in a pipe, *Proc. Roy. Soc. (London)* A223 (1954) 446–468.
- [17] M.L. Hunt, C.L. Tien, Effects of thermal dispersion on forced convection in fibrous media, *Int. J. Heat Mass Transfer* 31 (1988) 301–309.
- [18] O.A. Plumb, The effect of thermal dispersion on heat transfer in packed bed boundary layers, *Proc. ASME JSME Thermal Engineering Joint Conference* 2 (1983) 17–22.
- [19] D. Gidaspow, *Multiphase Flow and Fluidization — Continuum and Kinetic Theory Descriptions*, Academic Press, San Diego, 1994.
- [20] M. Jamialahmadi, M.R. Malayeri, H. Mueller-Steinhagen, Prediction of heat transfer to liquid–solid fluidized beds, *Canadian J. of Chemical Engineering* 73 (1995) 444–455.
- [21] Y. Kato, K. Uchida, T. Kago, S. Morooka, Liquid holdup and heat transfer coefficient between bed and wall in liquid–solid and gas–liquid–solid fluidized beds, *Powder Technology* 28 (1981) 173–179.